Name.....ID.....Section.....1.....Seat No.....

Sirindhorn International Institute of Technology
Thammasat University

Midterm Examination: Semester 2 / 2017

Course Title: ECS452 Digital Communication Systems

Instructor: Asst. Prof. Dr.Prapun Suksompong

Date/Time: March 16, 2018 / 9:00-11:00

Instructions:

- This examination has .....8..... pages (including this cover page).
- Conditions of Examination:

☐ Open book	☐ No dictionary	☐ No calculator	☐ Calculator allowed
☐ Closed book	☐ No dictionary	☐ No calculator	☐ Calculator allowed
☑ Semi-Closed book (.....1.....sheet(s)	☑ 1 page	☐ both sides of A4 paper note)	
These sheets must be hand-written. They should be **submitted with the exam**. Do not modify (,e.g., add/underline/highlight) content on the sheet inside the exam room. Indicate your name and ID in the upper-right corner of the sheet (in portrait orientation).			
	☐ No dictionary	☐ No calculator	☑ Calculator allowed
☐ Other:.....			

- **Read these instructions and the questions carefully**.
- Students are not allowed to be out of the examination room during examination. Going to the restroom may result in score deduction.
- Turn off all communication devices (mobile phone, etc.) and place them with other personal belongings in the area designated by the proctors or outside the test room.
- Write your name, student ID, and seat number clearly in the spaces provided on the top of this sheet. Then, write your **first name and the last three digits of your ID** in the spaces provided on the top of each page of your examination paper, starting from page 2.
- The examination paper is not allowed to be taken out of the examination room. Also, do not remove the staple. Violation may result in score deduction.
- Unless instructed otherwise**, *write down all the steps*** that you have done to obtain your answers.
  - When applying formula(s), state clearly which formula(s) you are applying before plugging-in numerical values.
  - You may not get any credit even when your final answer is correct without showing how you get your answer.
  - Formula(s) not discussed in class can be used. However, derivation must also be provided.
  - **Exceptions**:
    - Problems that are labeled with "**ENRPr**" (Explanation is not required for this problem.)
    - Parts that are labeled with "**ENRPa**" (Explanation is not required for this part.)

    These problems/parts are graded solely on your answers. There is no partial credit and it is not necessary to write down your explanation. Usually, spaces (boxes or cells in a table or rows of dashes) will be provided for your answers. "WACSP" stands for "write your answer(s) in the corresponding space(s) provided".
- The back of each page will not be graded; it can be used for calculations of problems or parts that do not require explanation (indicated by the label ENRPr or ENRPa).
- When not explicitly stated/defined, all notations and definitions follow ones given in lecture.
- Some points are reserved for *accuracy* of the answers and also for reducing answers into their *simplest* forms. Watch out for roundoff error. The error in your final answer should not exceed 0.1%.
- Points marked with * indicate challenging problems.
- Do not cheat. Do not panic. **Allocate your time wisely**.
- Don't forget to submit your fist online self-evaluation form by the end of today.

Problem 1. (10 pt) [ENRPr] Consider two codes (for source coding without extension) below.

Codebook for Code A:

x	e	ℓ	m	n	o
$c(x)$	101	110	111	011	100

Codebook for Code B:

x	e	ℓ	m	n	o
$c(x)$	0	100	1010	1011	11

Is Code A nonsingular?

Is Code B nonsingular?

Is Code A prefix-free?

Is Code B prefix-free?

Is Code A a Huffman code for some DMS?

Is Code B a Huffman code for some DMS?

The string 110101111100011 is encoded by Code A.
Decode it.

The string 10100100111011 is encoded by Code B.
Decode it.

Find the expected codeword length when Code A is used to encode a DMS with the following probabilities:

x	e	ℓ	m	n	o
$p_X(x)$	0.1	0.1	0.2	0.2	0.4

Find the expected codeword length when Code B is used to encode a DMS with the following probabilities:

x	e	ℓ	m	n	o
$p_X(x)$	0.1	0.1	0.2	0.2	0.4

Problem 2. (3 pt) Consider two codes (for source coding) below.

Codebook for Code C:

x	e	ℓ	m	n	o
$c(x)$	00	01	010	011	1100

Codebook for Code D:

x	e	ℓ	m	n	o
$c(x)$	0	001	0101	1101	11

(a) (1* pt) Is code C uniquely decodable? If yes, explain how to decode any encoded string. If no, give an example of an encoded string that is not uniquely decodable.

(b) (1 pt) [ENRPa] Is code D uniquely decodable?

(c) (1 pt) [ENRPa] Is code D instantaneous?

Problem 3. (10 pt) Consider a random variable X which has five possible values. Their probabilities are shown in the table below.

x	$p_X(x)$	Codeword $c(x)$	$\ell(x)$
e	0.42		
ℓ	0.17		
m	0.08	1011	
n	0.08		
0	0.25		

(a) (3 + 2 + 1 = 6 pt) Find a binary Huffman code (without extension) for this X. Put the values of the codewords and the codeword lengths in the table. Note that the codeword for the source symbol "m" is required to be 1011.

(b) (2 pt) Find the expected codeword length when Huffman coding is used without extension.

(c) (2 pt) Find $H(X)$.

Problem 4. (7 pt) A memoryless source emits two possible messages Y(es) and N(o) with probabilities 0.4 and 0.6, respectively.

(a) (2+2+1 = 5 pt) Find the codeword lengths when Huffman binary code with second-order extension is used to encode this source. Put the values of the corresponding probabilities and the codeword lengths in the table below.

x_1x_2	$p_{X_1,X_2}(x_1,x_2)$	$\ell(x_1,x_2)$
YY		
YN		
NY		
NN		

(b) (2 pt) Find L_2. (This is the expected codeword length per source symbol of the Huffman binary code for the second-order extension of this source.)

Problem 5. (13 pt) [ENRPr] Consider a DMC whose transition matrix $\mathbf{Q}$ is

$$\begin{array}{c|cccc} x \setminus y & 1 & 2 & 3 & 4 \\ 1 & 0.3 & 0.4 & 0.2 & 0.1 \\ 2 & 0.2 & 0.5 & 0.1 & 0.2 \\ 3 & 0.1 & 0.3 & 0.3 & 0.3 \end{array}$$

(a) (12 pt) Suppose the input probability vector is $\underline{\mathbf{p}} = [0.4, 0.5, 0.1]$.

In this problem, we will consider three decoders:

(i) (5 pt) The MAP decoder.

(ii) (5 pt) The ML decoder.

(iii) (2 pt) The naive decoder defined by $\hat{x}_{\text{naive}}(y) = y$.

Find the decoding table for each of these decoders. Also, find their probabilities of decoding error. Put your answers in the table below.

y	$\hat{x}_{\text{MAP}}(y)$	$\hat{x}_{\text{ML}}(y)$	$\hat{x}_{\text{naive}}(y)$
1			
2			
3			
4			
$P(\mathcal{E})$			

(b) (1 pt) Suppose the input probability vector is $\underline{\mathbf{p}} = [1/3, 1/3, 1/3]$. Find the error probability of the MAP decoder (which may not be the same as the decoder found in part (a)).

Problem 6. (3 pt) [ENRPr] Consider a BAC whose $Q(1|0) = 0.3$ and $Q(0|1) = 0.1$. Suppose $P[X = 0] = 0.2$. Draw the channel diagram. Then, find the probability transition matrix $\mathbf{Q}$ and the joint pmf matrix $\mathbf{P}$.

Channel Diagram	**Q** matrix	**P** matrix

Problem 7. (9 pt) [ENRPr] For each of the following DMC's probability transition matrices $\mathbf{Q}$,

(i) (2 pt) is the corresponding DMC symmetric? (Yes or No)

(ii) (2 pt) is the corresponding DMC weakly symmetric? (Yes or No)

(iii) ($5 = 2 + 2 + 0.5^* + 0.5^{**}$ pt.) calculate the corresponding capacity value [bpcu]. (Your answer should be of the form X.XXXX.)

Q	Symm.?	Weakly Symm.?	C
$\begin{bmatrix} 1/2 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1/2 & 0 & 1/2 & 0 \end{bmatrix}$			—·————
$\begin{bmatrix} 0 & 0 & 1/2 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \\ 1/2 & 1/2 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 \end{bmatrix}$			—·————
$\begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix}$			—·————
$\begin{bmatrix} 1/2 & 1/2 \\ 0 & 1 \end{bmatrix}$			—·————

Problem 8. (4 pt) A channel encoder maps blocks of two bits to seven-bit (channel) codewords. The four possible codewords are 0110010, 1110110, 1011010, and 1110111. A codeword is transmitted over a BSC whose crossover probability $p = 0.4$. Suppose the receiver observes $\underline{\mathbf{y}} = 0101010$ at the output of the BSC.

(a) (1 pt) Find the code rate.

(b) (2 pt) Assume that all four codewords are equally likely to be transmitted. Given the observed 0101010 at the receiver, what is the most likely codeword that was transmitted?

(c) (1* pt) [ENRPa] Assume that the four codewords are not equally likely. Their prior probabilities are 0.1, 0.2, 0.3, and 0.4, respectively.

Given the observed 0101010 at the receiver, what is the most likely codeword that was transmitted?

Problem 9. (7 pt) [ENRPr] Consider a pair of random variables X and Y whose joint pmf matrix $\mathbf{P}$ is given by

$$\begin{array}{c|cccc} x \setminus y & 1 & 2 & 3 & 4 \\ 1 & 1/8 & 0 & 1/8 & 0 \\ 2 & 1/8 & 1/8 & 0 & 0 \\ 3 & 0 & 1/8 & 1/8 & 1/4 \end{array}$$

Calculate the quantities in the table below.

$H(X,Y)$	
$H(X)$	
$H(Y)$	
$H(Y\|X=3)$	
$H(Y\|X)$	
$H(X\|Y)$	
$I(X;Y)$	

Problem 10. (1 pt)

(a) (1 pt) Do not forget to submit your study sheet with your exam.

(b) Reminder:

(i) Make sure that you write your name and ID on every page. (Read the instruction on the cover page.)

(ii) The online self-evaluation form is due by the end of today.